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Experimental Test of the Variability of G Using Viking Lander Ranging Data

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Results are presented from the analysis of solar system astrometric data, notably the range data to the Viking landers on Mars. A least-squares fit of the parameters of the solar system model to these data limits a simple time variation in the effective Newtonian gravitational constant to $(0.2 \pm 0.4) \times 10^{-11}$ year^{-1} and a rate of drift of atomic clocks relative to the implicit clock of relativistic dynamics to $(0.1 \pm 0.8) \times 10^{-11}$ year^{-1}. The error limits quoted are the result of uncertainties in the masses of the asteroids.

1. INTRODUCTION

In the last few decades, several theories have been proposed in which there is predicted a drift in the orbital periods of the planets relative to the time kept by atomic clocks (Brans and Dicke, 1961; Bergmann, 1968; Will and Nortvedt, 1972; Hellings and Nortvedt, 1973; Rosen, 1974; Canuto *et al.*, 1977; Adams, 1983; Canuto and Goldman, 1982; Dirac, 1979). These theories have been motivated, in large part, by a desire to incorporate Dirac's (1937) large numbers hypothesis into physics, tying the values of local physical constants to the evolution of the universe. One therefore expects the rate of the drift to be the inverse of the Hubble time, or $\sim 5 \times 10^{-11}$ year^{-1}.

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As we discuss below, there are two ways in which this period drift may be produced. Either cosmic effects may couple directly to gravitational physics, producing a time-dependent renormalization of the Newtonian gravitational constant G (Brans and Dicke, 1961; Bergmann, 1968; Will and Nortvedt, 1972; Hellings and Nortvedt, 1973; Rosen, 1974), or they may be coupled directly to atomic physics, causing atomic clocks to drift at a rate $\dot{\xi}$ relative to the implicit clock of relativistic dynamics (Canuto *et al.*, 1977; Adams, 1983; Canuto and Goldman, 1982; Dirac, 1979). We point out that in $\dot{\xi}$ theories (Canuto *et al.*, 1977; Adams, 1983; Canuto and Goldman, 1982; Dirac, 1979) there is a time variation induced in GM, since these theories generally require $GM\xi = \text{const}$. Therefore, all quoted limits on $\dot{\xi}$ may be interpreted as limits on $(GM)^{\cdot}/(GM)$ as well. Using all available solar system astrometric data, notably the range data to the Viking landers on Mars, we limit the values of $\dot{G}/G$ and $\dot{\xi}$ to

$$\dot{G}/G = (0.2 \pm 0.4) \times 10^{-11}\ \text{year}^{-1} \tag{1a}$$

$$\dot{\xi} = (0.1 \pm 0.8) \times 10^{-11}\ \text{year}^{-1} \tag{1b}$$

The quoted errors in equation (1) are much larger than the formal standard deviations and represent uncertainties stemming from our lack of knowledge of the masses of the asteroids, as discussed below. This sensitivity represents more than an order-of-magnitude improvement over previous limits set with radar ranging data (Reasenberg and Shapiro, 1978) or with lunar orbit data (Van Flandern, 1981).

2. THEORY

As pointed out in recent papers (Adams *et al.*, 1983; Canuto *et al.*, 1984), there are at least two conceptually different ways in which an incommensurability between orbital periods and atomic times might be produced. One way (Brans and Dicke, 1961; Bergmann, 1968; Will and Nortvedt, 1972; Hellings and Nortvedt, 1973; Rosen, 1974; Adams *et al.*, 1983, Canuto *et al.*, 1984) is to modify gravitational physics by postulating additional cosmic fields which affect the form of the physical metric tensor (i.e., the geometry measured by atomic rods and clocks). The Brans and Dicke (1961) theory is typical of such a theory. In these theories, the drift in the orbital periods results from a time-dependent renormalization of the gravitational constant due to the slowly changing values of the cosmic fields. One writes the effective gravitational constant as

$$G_{\text{eff}} = G[1 + (\dot{G}/G)(t - t_0)] \tag{2}$$

where G and $\dot{G}$ are the values of the gravitational constant and its time derivative at some $t = t_0$. With this G_{eff} in the metric, there appears a term in the geodesic equation which gives rise to a coordinate acceleration

$$\delta a^i = -\frac{\dot{G}}{G}\left[\frac{GM}{r}\frac{x^i}{r^2}(t-t_0)\right] \tag{3}$$

in addition to the usual Newtonian and relativistic terms.

A second way (Canuto *et al.*, 1977; Adams, 1983; Canuto and Goldman, 1982; Dirac, 1979; Adams *et al.*, 1983) in which an incommensurability between atomic times and gravitational periods might be produced would be for there to exist some cosmic influence which directly affected atomic physics while the field equations of gravitation remained unchanged. In this case, there would be an observable effect of the clock drift on solar system measurements because the time associated with each astrometric observation is the time given by an atomic clock. The form of the effect on the coordinate acceleration equations is obtained by first defining a parameter $\dot{\xi}$ which gives the rate at which atomic time (dt) will drift away from the proper time inherent in relativistic dynamics (called Einstein time dt_E),[5]

$$dt_E/dt = 1 + \dot{\xi}(t - t_0) \tag{4}$$

After transforming the geodesic equation to reflect this new definition of the affine parameter, one obtains a nongeodesic equation of motion

$$\frac{d^2x^\mu}{dt^2} + \Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{dt}\frac{dx^\beta}{dt} = \dot{\xi}\left(g^{\mu 0} - \frac{dx^\mu}{dt}\frac{dx^0}{dt}\right) \tag{5}$$

where the metric is that measured by atomic rods and clocks and $\Gamma^\mu_{\alpha\beta}$ are the Christoffel symbols derived from that metric. There is thus in the coordinate acceleration an additional term springing from the right-hand side of equation (5):

$$\delta a^i = \dot{\xi}\left[\frac{GM}{r}\frac{x^i}{r^2}(t-t_0) - \frac{dx^i}{dt}\right] \tag{6}$$

In order to determine the values of $\dot{G}/G$ and $\dot{\xi}$ by means of the Earth–Mars data, equations (3) and (6) were numerically integrated to get the perturbations which each parameter would produce in the Earth–Mars range. The

[4] $\dot{\xi}$ has been variously called $\dot{\beta}$, $\dot{\beta}_a$, and $\dot{\varphi}$ in Canuto *et al.* (1977), Adams (1983), Canuto and Goldman (1982), Dirac (1979), and Reasenberg and Shapiro (1978).

signature of each perturbation was then searched for in the data. Comparison of equations (3) and (6) shows that the two parameters will differ in their effects, a fact which can be exploited in order to discriminate observationally between the two categories of theories of cosmic influence on local physics.

3. SOLAR SYSTEM DATA AND MODEL

Beginning in July 1976 and ending in July 1982, 1136 range measurements between tracking stations of the Deep Space Network and the Viking landers on Mars were taken with an average interval between measurements of about 2 weeks. The range measurement itself is accurate to about 2 m, while uncertainties in the time delay produced by interplanetary plasma, along with unknown delays inherent in the tracking station and spacecraft signal paths, leave a calibration uncertainty of about 9 m. In order to fit the data, the Earth's orbit must be adjusted along with the orbit of Mars. However, the Earth's motion is also constrained by astrometric observations of the other bodies of the solar system. It has therefore proven necessary to include all reliable solar system data and all relevant solar system parameters in adjusting the solar system model to fit the Viking data. The data used are (1) 1136 range measurements to the Viking landers on Mars (from July 1976 to July 1982), (2) 645 range measurements to the Mariner 9 spacecraft in orbit around Mars (from November 1971 to October 1972), (3) 1305 radar bounce range measurements from the surfaces of Mercury and Venus (from 1964 to 1977), (4) 2954 lunar laser range measurements (from 1969 to 1980), (5) 44,755 optical position measurements of the sun and planets, right ascension and declination (from 1911 to 1979). The parameters being solved for include (1) initial orbital elements of the moon and planets, (2) the masses of Venus, Earth, and the outer planets, (3) the masses of the three asteroids (Ceres, Pallas, and Vesta) which have a large effect on Mars' orbit, (4) the average densities of the remaining asteroids, divided into two groups according to compositional type, (5) the parameters $\dot{\xi}$ and $\dot{G}/G$, and (6) other parameters representing corrections for systematic effects known to exist in individual data sets or parameters unique to each data set.

These parameters were fitted to the data in a least-squares sense, resulting in estimates and standard deviations for each of the parameters. If all errors in the data sets were random Gaussian-distributed errors, then the formal standard deviation given by the least-squares fit would be the correct estimate of the uncertainty in the values of the parameters. Experience has shown, however, that there typically exist small unmodeled errors in the data sets, and that additional care is required to estimate the realistic uncertainty that should be assigned to the values.

The major uncertainty in the determination of $\dot{G}/G$ and $\dot{\xi}$ comes from uncertainty in the masses of the asteroids. We determined that the three largest asteroids (Ceres, Pallas, and Vesta) had large periodic effects in the Earth–Mars range which could be separated, allowing their masses to be determined in the solutions. The 200 largest remaining asteroids in the belt which had fairly well-determined radii (Tedesco, 1983) were divided into two groups. The C group consisted of asteroids generally in the outer regions of the belt having compositional type C, F, P, and D. The S group contained asteroids of type S, M, E, and R which are found in greatest abundance in the inner regions of the belt (Chapman, 1978). The asteroids in each group were treated as point masses with a common density (to be determined in the solutions) and with volumes determined from their measured diameters. This model of the asteroid belt, three individual masses plus two groups with uniform density, was found to be adequate for our purposes in that (1) the model fitted the data well enough to eliminate visible trends in the residuals, and (2) solving for the masses of additional individual asteroids did not produce significant changes in the estimates of $\dot{G}/G$ or $\dot{\xi}$.

4. RESULTS

The results are displayed in Table I. Two sets of solutions are shown, one set for $\dot{G}/G$ and one set for $\dot{\xi}$. In each set, the numbers listed under Run A are the values obtained when the densities of the two asteroid groups were solved for. In these solutions, the density of the C group of asteroids was found to be uncertain by about 30% and turned out to have a larger value than had been expected. We have considered the possibility that this large value may be due to a small systematic error in the data and have, in

Table I. Sample Solutions[a]

Parameter	Run A	Run B	Run C	Run D
$\dot{G}/G$	2.8×10^{-12}	1.0×10^{-12}	-1.8×10^{-12}	5.5×10^{-12}
C density	3.4 ± 1.0	[2.0]	[1.5]	[3.5]
S density	2.7 ± 0.5	2.3 ± 0.3	[3.5]	[1.5]
$\dot{\xi}$	2.5×10^{-12}	-0.5×10^{-12}	-6.2×10^{-12}	8.8×10^{-12}
C density	3.1 ± 1.0	[2.0]	[1.5]	[3.5]
S density	2.6 ± 0.5	2.4 ± 0.3	[3.5]	[1.5]

[a] Units of $\dot{G}/G$ and $\dot{\xi}$ are year^{-1} and density units are g/cm^3. See text for discussion.

Run B, fixed the value of the C-group asteroid density to be the $2\ \mathrm{g/cm^3}$ which was expected, while continuing to solve for the S-group density. The resulting values for $\dot{G}/G$ and $\dot{\xi}$ show the effect which would be produced if this systematic error were real. In Runs C and D the values of the densities were set *a priori* equal to the values in brackets. These cases represent what we feel are extreme but reasonable values for both densities and were chosen for display because they produced extreme values in $\dot{G}/G$ or $\dot{\xi}$ without producing noticeable trends in the data residuals. Table I represents only a sampling of the over 100 solutions which were made in the course of this investigation. In each case, for $\dot{G}/G$ and for $\dot{\xi}$, our quoted best value in equation (1) is the arithmetic mean between the solutions in Run A and Run B. The uncertainties we quote in equation (1) come from the extreme values derived in Runs C and D. All other solutions had values within these limits.

We have also assessed the effects of uncertainties in the observed radii of the asteroids. The nominal radius values we used were taken from the most recent compilation by Tedesco (1983). Any systematic errors in the radii for a given group would just scale the size of their effect, and would thus be absorbed into a new value for the density. This value would be in error, but it would not affect the estimates of $\dot{G}/G$ or $\dot{\xi}$. The effects of random errors in the radii were investigated by a Monte Carlo technique of randomly adjusting the radii of the 200 asteroids by amounts consistent with their known uncertainties (about 10% for well-observed asteroids and 50% for asteroids which have not been well observed). The resulting range of values for $\dot{G}/G$ and $\dot{\xi}$ fell well within the error limits quoted in equation (1). In addition, the effect of the 2000+ asteroids which were not included in our two groups was found to be negligible, producing a correction to the modeled asteroid effect of less than 5%.

It was also determined that, while the Mariner 9 data and the lunar laser ranging data could be taken in and out of the fit without producing any major discrepancy in the results, the accuracy of the estimates of $\dot{G}/G$ or $\dot{\xi}$ was severely degraded without the Viking data.

Finally, several solutions were made in which $\dot{G}/G$ and $\dot{\xi}$ were determined simultaneously. The values derived in these cases were consistent with zero for both parameters, though the uncertainty increased to $\pm 2.1 \times 10^{-11}\ \mathrm{year}^{-1}$ for $\dot{G}/G$ and $\pm 4.2 \times 10^{-11}\ \mathrm{year}^{-1}$ for $\dot{\xi}$. These results indicate that neither category of theories of cosmic influence on local physics seems to be preferred at this level of sensitivity.

We conclude that the available solar system astrometric data severely limit the existence of a cosmic influence on either gravitational ($\dot{G}/G$) or nongravitational ($\dot{\xi}$) local physics at the level expected from Dirac's large numbers hypothesis.

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